A NEW METHOD OF SOLVING THE INTEGRAL EQUATIONS OF RADIATION HEAT TRANSFER

S. M. Bednov

UDC 536.3

We present a method for calculating the radiation heat transfer between bodies and a new approach to the solution of the corresponding integral equations. We show, in fact, that the first approximation describes the solution of these equations with sufficiently high accuracy and is, in some cases, exact.

For simplicity we consider the problem of calculating the radiation heat transfer between infinite parallel isothermal strips with identical radiation characteristics in the very same situation as in [1] (see Fig. 1). The calculation reduces to the determination of the effective radiation thermal flow profiles over the surfaces of the strips from the following system of integral equations:

$$\begin{aligned} E_{\text{eff}1} &= \varepsilon\sigma T_1^4 + (1-\varepsilon)\int_0^1 E_{\text{eff}2}(y)K(x,\ y,\ \gamma)\,dy, \\ E_{\text{eff}2} &= \varepsilon\sigma T_2^4 + (1-\varepsilon)\int_0^1 E_{\text{eff}1}(x)K(x,\ y,\ \gamma)\,dx, \end{aligned} \tag{1}$$

where

$$K(x,\ y,\ \gamma) = \frac{\gamma^2}{2\,[(x-y)^2+\gamma^2]^{3/2}}.$$

It was shown in [1-3] that following appropriate transformations the solution can be found in a generalized form depending on dimensionless parameters of the given problem. When $T_1 \neq T_2$ the solution is obtained as the sum of two solutions: 1) for equal temperatures of the strips, and 2) when one strip has zero temperature and the other has the temperature $T^* = (T_2^4 - T_1^4)^{1/4}$ $(T_2 > T_1)$. For the first case we solve the equation

$$\beta_a(x) = 1 - (1-\varepsilon)\int_0^1 \beta_a(y)K(x,\ y,\ \gamma)\,dy, \tag{2}$$

for the second case we solve the system of integral equations:

$$\begin{aligned} \beta_{B_1}(x) &= (1-\varepsilon)\int_0^1 \beta_{B_2}(y)K(x,\ y,\ \gamma)\,dy, \\ \beta_{B_2}(y) &= 1 + (1-\varepsilon)\int_0^1 \beta_{B_1}(x)K(x,\ y,\ \gamma)\,dx, \end{aligned} \tag{3}$$

where

$$\beta_a = \frac{E_{\text{eff}_a}(x)}{\varepsilon\sigma T_1^4};\quad \beta_{B_1} = \frac{E_{\text{eff}\,1}(x)}{\varepsilon\sigma T^{*4}};\quad \beta_{B_2} = \frac{E_{\text{eff}\,2}(x)}{\varepsilon\sigma T^{*4}}.$$

Knowing the solutions of Eq. (2) and system (3), we obtain the profiles of the output thermal flows over the strips from the relations

$$q_i(x) = \varepsilon\sigma T_2^4\,\frac{1-\varepsilon\beta_a(x)}{1-\varepsilon} + \varepsilon\sigma T^{*4}\beta_i(x),\quad i = 1,\ 2, \tag{4}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 6, pp. 1031-1040, December, 1986. Original article submitted September 26, 1985.

0022-6841/86/5106-1485$12.50

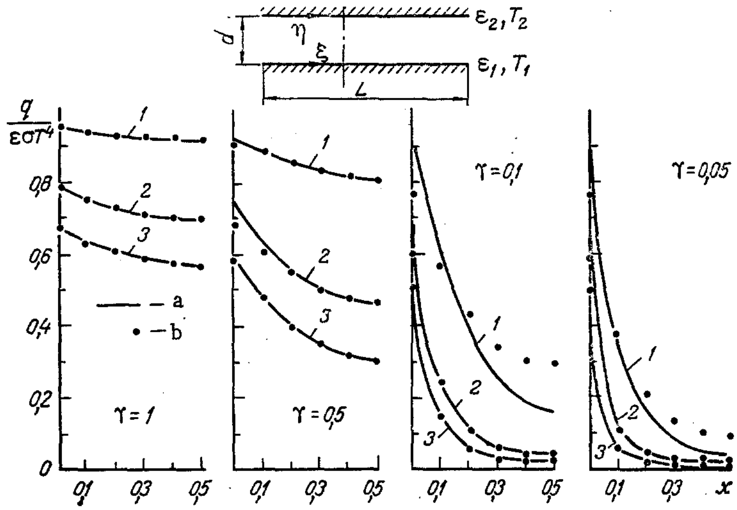

Fig. 1. Thermal flow profiles for parallel strips ($\varepsilon_1 = \varepsilon_2 = \varepsilon$, $T_1 = T_2 = T$): a) by formula (27); b) numerical solution [1]; curves labeled 1, 2, 3 are for ε values 0.1, 0.5, 0.9.

where

$$\beta_1(x) = -\frac{\varepsilon}{1-\varepsilon}\beta_{\text{B}_1}(x);\quad \beta_2(x) = \frac{1-\varepsilon\beta_{\text{B}_2}(x)}{1-\varepsilon}.$$

This method, however, can be simplified somewhat. The initial equations are linear in T^4; therefore we seek a solution of system (1) in the form

$$E_{\text{eff}_i}(x) = \varepsilon\sigma T_1^4 f_{2i-1}(x) + \varepsilon\sigma T_2^4 f_{2i}(x),\ i = 1,\ 2. \tag{5}$$

It follows from Eqs. (1) that $f_1(x) \equiv f_4(x)$; $f_2(x) \equiv f_3(x)$. Functions $f_1(x)$ and $f_2(x)$ may be obtained from a system analogous to system (3):

$$\begin{aligned} f_1(x) &= 1 + (1-\varepsilon)\int_0^1 f_2(y)K(x,\ y,\ \gamma)\,dy, \\ f_2(y) &= (1-\varepsilon)\int_0^1 f_1(x)K(x,\ y,\ \gamma)\,dx. \end{aligned} \tag{6}$$

The thermal flow profiles are then

$$q_i(x) = \frac{1-\varepsilon f_1(x)}{1-\varepsilon}\varepsilon\sigma T_i^4 - \frac{\varepsilon f_2(x)}{1-\varepsilon}\varepsilon\sigma T_{3-i}^4,\ i = 1,\ 2. \tag{7}$$

It follows from this that the solution of the general problem for $T_1 \neq T_2$ can only be found from the system (3). When $T_1 = T_2$ the thermal flow distribution function is $\beta_a(x) = f_1(x) + f_2(x)$. This simplifies the solution of the initial problem since there is no need to solve the integral Eq. (2).

In [1-3] systems of two bodies were considered with identical radiation properties. However, a solution can be found even for $\varepsilon_1 \neq \varepsilon_2$. We seek such a solution in the form

$$E_{\text{eff}_i}(x) = \varepsilon_i\sigma T_i^4\varphi_{i^2}(x) + \varepsilon_{3-i}\sigma T_{3-i}^4\sqrt{\frac{1-\varepsilon_i}{1-\varepsilon_{3-i}}}\varphi_{i+1}(x),\ i = 1,\ 2. \tag{8}$$

It is easy to show that $\varphi_1(x) \equiv \varphi_4(x)$ and $\varphi_2(x) \equiv \varphi_3(x)$, while the functions $\varphi_1(x)$ and $\varphi_2(x)$ are determined from the following system of integral equations:

$$\varphi_1(x) = 1 + \sqrt{(1-\varepsilon_1)(1-\varepsilon_2)}\int_0^1 \varphi_2(y)K(x,\ y,\ \gamma)\,dy, \tag{9}$$

$$\varphi_2(y) = \sqrt{(1-\varepsilon_1)(1-\varepsilon_2)} \int_0^1 \varphi_1(x) K(x,\ y,\ \gamma)\, dx.$$

The thermal flows are calculated from the relations

$$q_i(x) = \varepsilon_i \sigma T_i^4 \frac{1-\varepsilon_i \varphi_1(x)}{1-\varepsilon_i} - \varepsilon_{3-i}\, \sigma T_{3-i}^4\, \frac{\varepsilon_{3-i}\, \varphi_2(x)}{\sqrt{(1-\varepsilon_1)(1-\varepsilon_2)}},\ i = 1,\ 2. \tag{10}$$

Thus, we see that the solution (6) for $\varepsilon_1 = \varepsilon_2$ contains as a special case the solutions of the more general problem for $\varepsilon_1 \neq \varepsilon_2$. Actually, the coefficients of the integral terms in systems (6) and (9) vary from 0 to 1 and solutions with identical coefficients coincide. For example, for $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.9$ the quantity $\sqrt{(1-\varepsilon_1)(1-\varepsilon_2)} = 0.3$ and the functions $\varphi_1(x)$ and $\varphi_2(x)$ from system (9) are identically equal, respectively, to functions $f_1(x)$ and $f_2(x)$ from system (6) for $\varepsilon = 0.7$.

For completeness we consider the case in which one or both of the strips is black. Here the effective flow profiles are constant over the surface of the plates; however, the profiles of the exiting thermal flows vary since each point on the surface of a strip "sees" the surrounding "cold" "black" space at its own solid angle, and the given problem is solved in terms of the incident flows. For definiteness, let $\varepsilon_1 = 1$; then the initial system of equations becomes

$$E_{\text{in}1}(x) = \int_0^1 E_{\text{eff}_2}(y) K(x,\ y,\ \gamma)\, dy,\ \ E_{\text{eff}_2}(y) = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\, \sigma T_1^4 z(y), \tag{11}$$

where

$$z(y) = \int_0^1 K(x,\ y,\ \gamma)\, dx.$$

The thermal flow profiles are then, respectively,

$$q_1(x) = \sigma T_1^4 [1 + (1-\varepsilon_2)\, \omega(x)] - \varepsilon_2 \sigma T_2^4 z(x),\ \ q_2(x) = \varepsilon_2 \sigma T_2^4 - \varepsilon_2 \sigma T_1^4 z(x), \tag{12}$$

where

$$\omega(x) = \int_0^1 z(y) K(x,\ y,\ \gamma)\, dy,$$

in which case the solution may be obtained in quadratures. These same relations apply even when both of the strips are black.

Next, we consider the very same problem but in a somewhat modified setting. Assume now that the strips $\varepsilon_1 = \varepsilon_2 = \varepsilon$ have constant thermal emission and are not maintained at constant temperatures. Since this problem was not considered earlier in the literature, we investigate its solution in some detail. In this case the initial system of equations is analogous to the system (1), except that T is a function of the coordinates. The heat balance at each point of the strips can be written as follows:

$$q_i = \varepsilon \sigma T_i^4(x) - \varepsilon E_{\text{in}\, i}(x),\quad i = 1,\ 2. \tag{13}$$

Let $\varepsilon \neq 1$; writing E_{in} in terms of E_{eff}, from system (1) we obtain

$$E_{\text{eff}_i}(x) = q_i + \int_0^1 E_{\text{eff}_{3-i}}(y) K(x,\ y,\ \gamma)\, dy,\ i = 1,\ 2. \tag{14}$$

Using the principle of superposition, we seek a solution in the form

$$E_{\text{eff}\,i}(x) = q_1 \theta_{2i-1}(x) + q_2 \theta_{2i}(x),\ i = 1,\ 2. \tag{15}$$

Similar to what was done before, we can show that $\theta_1(x) \equiv \theta_4(x)$, $\theta_2(x) \equiv \theta_3(x)$. The functions $\theta_1(x)$, $\theta_2(x)$ are obtained from the following system of integral equations:

$$\begin{aligned} \theta_1(x) &= 1 + \int_0^1 \theta_2(y) K(x,\ y,\ \gamma)\, dy, \\ \theta_2(y) &= \int_0^1 \theta_1(x) K(x,\ y,\ \gamma)\, dx. \end{aligned} \tag{16}$$

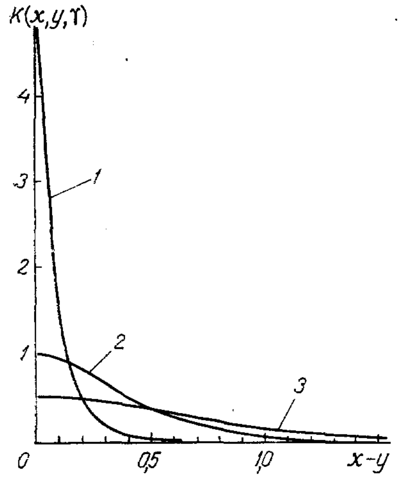

Fig. 2

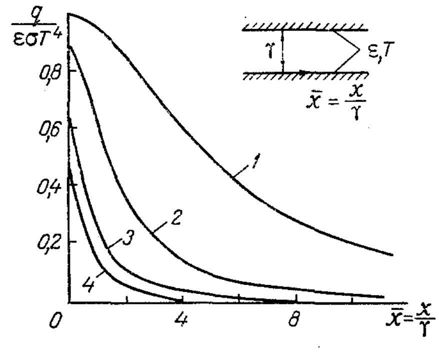

Fig. 3

Fig. 2. Dependence of K(x, y, γ) on x–y for various values of the parameter γ: curves labeled 1, 2, and 3 are for γ values of 0.1, 0.5, and 1, respectively.

Fig. 3. Distribution of thermal flows in a boundary zone for a set of two plates: curves labeled 1, 2, 3, and 4 are for ε values of 0.01, 0.1, 0.5 and 1, respectively.

Thus we see that the solution in this case is described by an analogous system, just as in the case of constant temperatures of the strips [system (6)], except that here the coefficient of the integral terms is equal to 1; therefore, the functions $\theta_1(x)$ and $\theta_2(x)$ depend on a single parameter γ. The temperatures are calculated from the relations

$$T_i(x) = \left[q_i \frac{1-\varepsilon(1-\theta_1(x))}{\varepsilon\sigma} + q_{3-i}\frac{\theta_2(x)}{\varepsilon\sigma} \right]^{1/4}, \; i=1,\, 2. \tag{17}$$

If there is equal thermal emission on the plates, then

$$T_1(x) \equiv T_2(x) \equiv T(x),$$

$$T(x) = \left[q\,\frac{1-\varepsilon+\varepsilon(\theta_1(x)+\theta_2(x))}{\varepsilon\sigma} \right]^{1/4}. \tag{18}$$

It is easy to show that when $\varepsilon_1 \neq \varepsilon_2$ we arrive at the very same relationships, except that in the terms in Eq. (17) containing q_1 we replace ε by ε_1 and in the terms containing q_2 we replace ε by ε_2. Introducing the equilibrium state temperature $T_{p_1} = (q_1/\sigma)^{1/4}$, we see that the physical meaning of the functions $\theta_1(x)$ and $\theta_2(x)$ is that they represent a dimensionless temperature to the fourth degree for the first and for the second strip when there is no heat emission from the second of the black strips:

$$\theta_1(x) = \left[\frac{T_1(x)}{T_{p_1}} \right]^{1/4}_{\varepsilon=1}, \quad \theta_2(x) = \left[\frac{T_2(x)}{T_{p_1}} \right]^{1/4}_{\varepsilon=1},$$

while the sum $\theta_1(x) + \theta_2(x)$ of the functions, as can be seen from the relations (18), is the dimensionless profile of the temperature to the fourth degree for black strips with identical heat emission.

Functions $\theta_1(x)$ and $\theta_2(x)$ can also be given a somewhat different interpretation. They represent dimensionless temperature profiles over the strips not only in the case $\varepsilon_1 = \varepsilon_2 = 1$ and $q_2 = 0$, but also when $\varepsilon_1 = 1$, $\varepsilon_2 \neq 1$, $q_2 = 0$. This is, of course, a consequence of the fact that the temperature profiles are determined by the internal heat emission of the "black" first strip. The "cold" strip ($q_2 = 0$) under stationary conditions radiates all that it absorbs. However, since the energies being radiated and absorbed are proportional to the degree of blackness of this strip, it is then obvious that the temperature profile on the strip is independent of its radiation properties. Moreover, the situation here in the heat transfer process is that the magnitude of the flow incident on the "hot" strip is identical in this and in the other case. In the first case the flow coming from the first strip is absorbed by the

TABLE 1. Values of the Total Thermal Flows in a System of Parallel Strips $Q/\varepsilon\sigma T^4L$

γ	$\varepsilon=0,1$			$\varepsilon=0,5$			$\varepsilon=0,9$		
	A	B	C	A	B	C	A	B	C
1	0,9340	0,9338	0,9338	0,7388	0,7384	0,7384	0,6010	0,6010	0,6010
0,5	0,8607	0,8576	0,8575	0,5538	0,5494	0,5490	0,4071	0,4064	0,4064
0,1	0,5122	0,422	0,365	0,17354	0,1593	0,149	0,1045	0,1029	0,1029
0,05	0,3388	0,252	0,205	0,09298	0,0826	0,082	0,05387	0,05278	0,0528

Comment: The data of column A were obtained using the "balance" method assuming constancy of the effective radiant fluxes over the whole surface of the strips [1]; column B data are from the numerical solution [1]; column C data reflect use of the proposed solution (27).

second and is radiated back onto the "hot" strip; in the second case, this flow incident on the first strip is composed of reradiated and rereflected flows, and it follows directly from the assumptions made that, both in magnitude as well as in their distribution in space, these flows incident on the first "hot" strip, both in the first as well as in the second case, are identical; therefore, the temperatures are also.

Thus, we have shown that the methodological solution of the problem of calculating the radiation heat transfer in a set of bodies under these circumstances can be reduced to solving a system of integral equations of the first kind of the type (6).

Usually (see [1-3]) these systems of equations are solved numerically or by the method of successive approximations [1-4] or by the method of "balance" [5]. However, a more detailed consideration of the kernel of the integral equations makes it possible to improve somewhat the convergence of the successive approximations and to propose an approximate solution of these equations in quadratures which, in many cases, may be represented in the form of analytic expressions.

Thus, for example, for the system of strips the kernel may be described by the function

$$K(x,\ y,\ \gamma)=\frac{\gamma^2}{2}\ \frac{1}{[(x-y)^2+\gamma^2]^{3/2}}\ . \tag{19}$$

This function possesses a singularity:

$$\lim_{\gamma\to 0} K(x,\ y,\ \gamma)=\lim_{\gamma\to 0}\frac{\gamma^2}{2}\ \frac{1}{[(x-y)^2+\gamma^2]^{3/2}}=\begin{cases}0, & x\neq y,\\ \infty, & x=y.\end{cases} \tag{20}$$

For clarity we exhibit the nature of the behavior of the function $K(x, y, \gamma)$ in its dependence on $x-y$ in Fig. 2 for various values of γ. The limit of the integral of this function along the coordinate axis as $\gamma \to 0$, excluding the boundary points, is equal to 1. At the boundary points its value is equal to 0.5. This is easy to verify by a direct integration or to determine by starting from physical considerations, since this integral is the total (dimensionless) thermal flow being radiated by elements on the strip into a halfspace (in this case onto the second strip since these strips are infinitely close together). It is possible to determine the magnitude of the flow at the boundary points from the following considerations. If we take two identical systems of two half planes, join them and form a system of two planes, then a boundary point, for definiteness on some lower half plane, turns out to be inside the system and the total thermal flow being radiated by it falls onto the upper plane. Since the upper half planes are identically situated relative to it, then, in like manner, the thermal flow radiated by this point of the lower half plane onto the upper half planes also contributes to each half plane exactly half of the total flow. Thus in the limit the function $K(x, y, \gamma)$ goes over into a δ-function [6]. Similar conclusions can also be made for a system of disks, planes, and, apparently, other systems. For clarity and simplification of our calculations we show how this can be used in solving, for example, the integral equation

$$p(x)=1+\rho\int_0^1 p(y)K(x,\ y,\ \gamma)\,dy. \tag{21}$$

There is no loss of generality in applying the method to the solution of Eq. (21), which describes the problems we have considered when $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $T_1 = T_2 = T$ or $q_1 = q_2 = q$. We rewrite this equation in the following equivalent form:

$$p(x)=\frac{1}{1-\rho\int\limits_0^1 K(x,\ y,\ \gamma)\,dy}\left[1+\rho\int\limits_0^1[p(y)-p(x)]\,K(x,\ y,\ \gamma)\,dy\right]. \tag{22}$$

We consider this equation in more detail. The second term in the brackets is small for large and small values of the parameter γ. Actually, for large γ the functions $p(x)$ and $p(y)$, which represent dimensionless effective radiation flows, are close to one another and vary weakly, the role of multiple reflections is small (the strips are located far from one another), and the difference $p(y) - p(x)$ is small. For small γ this term is also small since $K(x, y, \gamma)$ goes over into a δ-function. It follows from this that

$$\lim_{\gamma\to 0}\int\limits_0^1[p(y)-p(x)]\,K(x,\ y,\ \gamma)\,dy=\int\limits_0^1[p(y)-p(x)]\,\delta(x-y)\,dy=p(x)-p(x)\equiv 0. \tag{23}$$

Therefore, if we take as the solution of the initial integral Eq. (21) the first term in the Eq. (22), we find that such a solution is practically exact for large and small values of γ. For intermediate values of γ it will have an error, small for small ρ, which can be ascertained by comparison with a numerical solution. Moreover, using the description (22) for the initial integral Eq. (21), we can quickly obtain numerical solutions on a computer using the method of successive approximations. Indeed, if as the zeroth approximation we take the proposed approximate solution

$$p_0(x)=[1-\rho z(x)]^{-1}, \tag{24}$$

where $z(x)=\int\limits_0^1 K(x,\ y,\ \gamma)\,dy$, and calculate remaining approximations by the recursion formula

$$p_n(x)=p_0(x)\left[1+\rho\int\limits_0^1[p_{n-1}(y)-p_{n-1}(x)]\,K(x,\ y,\ \gamma)\,dy\right], \tag{25}$$

it is then clear that the following approximation is proportional to ρ^2 and not ρ, as in the direct use of this method for solving Eq. (21). We can give a physical clarification of this approximate solution. For small values of the rereflected flows the effective radiant fluxes are affected, in the main, by the radiation from the element in closest proximity. The contribution from this element is taken into account exactly while that from the other elements is accounted for approximately: in effect, these radiate in the direction of the element considered only as much as the element in closest proximity; however, since the local angular coefficient decreases sharply, the value itself of the incident flux from this is small, and this justifies such an assumption. For a small degree of blackness the rereflected flows are large and the effective radiation at the point considered is influenced by the large region on the other body, so that the relative error is large in comparison with the exact solution.

By way of example, we consider the calculation of the radiant heat transfer from a system of two strips [1]. As was shown in [1] for $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $T_1 = T_2 = T$, the thermal flow profiles may be calculated from the relation

$$\frac{q(x)}{\varepsilon\sigma T^4}=\frac{1-\varepsilon\beta_a(x)}{1-\varepsilon}, \tag{26}$$

where $\beta_a(x)$ is the solution of the integral Eq. (2). As the solution of this equation we take the function

$$\beta_0(x)=\frac{1}{1-(1-\varepsilon)\,z(x)}, \tag{27}$$

where

$$z(x)=\frac{\gamma^2}{2}\int\limits_0^1\frac{dy}{[(x-y)^2+\gamma^2]^{3/2}}=\frac{1}{2}\left[\frac{1-x}{\sqrt{(1-x)^2+\gamma^2}}+\frac{x}{\sqrt{x^2+\gamma^2}}\right].$$

The thermal flow profiles calculated with the aid of the relation (27) are shown in Fig. 1. The approximate solution coincides with the numerical solution for all ε for large γ and for all γ for large ε. The largest deviation is observed for $\varepsilon = 0.1$, where it is attained for intermediate values of γ. Thus, for $\gamma = 0.1$ this value is equal to 0.15; for $\gamma = 0.05$ it is 0.5, which, according to what was said earlier, should be the case. However, the relative value of the error for small ε is substantial, but it should be noted that the actual value of the thermal flow for small ε is small.

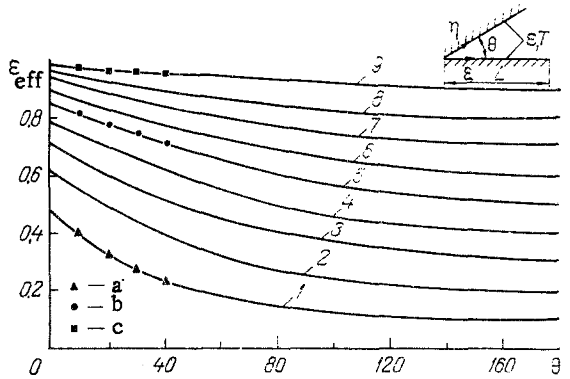

Fig. 4. Effective degree of blackness of a wedge-shaped cavity (points *a* are for ε = 0.1; points b are for ε = 0.5; points c are for ε = 0.9 and are the experimental data taken from [8]): curves labeled i, i = 1, 2, ..., 9, are for ε = i/10.

The value of the total thermal flows is found from the formula

$$\frac{Q}{\varepsilon\sigma T^4 L} = \int_0^1 q(x)\,dx. \tag{28}$$

In Table 1 we present the values of these flows taken from [1] and also their values calculated from the relation (27).

The total thermal flows are obtained in terms of quadratures; however, the integral (28) cannot be evaluated in closed form. The values of the flows were obtained numerically using Simpson's rule. As can be seen from Table 1 and Fig. 1, the approximate solution agrees very satisfactorily with the numerical solution [1] and is substantially more accurate than the results using the "balance" method. Moreover, taking into account that as $\gamma \to 0$ this solution tends towards the exact solution (whereas numerically this solution cannot be obtained by the usual method, as in [1]), we can study, fairly simply, the nature of its behavior in this case in the boundary zone of the two halfplanes. To this end, in the relation for z(x) we make the change of variables $\bar{x} = x/\gamma$, $\bar{y} = y/\gamma$ and let $\gamma \to 0$; then

$$z(\bar{x}) = \frac{1}{2}\int_0^\infty \frac{dy}{[(\bar{x}-\bar{y})^2-1]^{3/2}} = \frac{1}{2}\left(1 + \frac{\bar{x}}{\sqrt{\bar{x}^2+1}}\right), \tag{29}$$

and the limiting thermal flow profile becomes

$$\frac{q(\bar{x})}{\varepsilon\sigma T^4} = \frac{1}{1-\varepsilon}\left[1 - \varepsilon\frac{1}{1 - \frac{1-\varepsilon}{2}\left(1 + \frac{\bar{x}}{\sqrt{\bar{x}^2+1}}\right)}\right]. \tag{30}$$

The distribution of the thermal flows in this limiting case is shown in Fig. 3. The influence of the boundary effect for various ε is evident from these graphs. For large values of ε the radiation in the system of planes is practically closed off by 4 calibers from the edge; as ε decreases, the influence of the boundary zone becomes significantly stronger, and the radiation for ε = 0.1 is closed off by about 10 calibers. It is interesting to note that at a boundary point there exists a limiting value of the thermal flow below which it may not be dropped. From the relation (30) we have

$$\frac{q(0)}{\varepsilon\sigma T^4} = \frac{1}{1+\varepsilon}. \tag{31}$$

It is evident from Fig. 1 that for small values of the parameter γ the values of the flows at the boundary points do not appear below the limiting values in the data presented by the authors of [1]. This is apparently due to an error in their numerical solution. One can expect that if the numerical solution is carried out with greater precision the deviations among them will be somewhat less than in Fig. 1 and in Table 1.

The application of an analogous approach to the calculation of radiant heat transfer in the case $T_1 \neq T_2$ for other systems of bodies, as in [2, 3], as well as for the case $\varepsilon_1 \neq \varepsilon_2$ for problems with constant heat emission (these latter problems were solved by the author and also by the usual numerical method, for example, for systems of two plates) showed higher accuracy in the results obtained in comparison with numerical solutions (no more than 20% for the poorest of the versions considered).

We can also obtain a solution for a wedge-shaped cavity. The flow distribution in the same situation as that used in [1] was obtained in the form

$$\frac{E_{\text{eff}}(x)}{\varepsilon\sigma T^4}=\left\{1-\frac{1-\varepsilon}{2}\left[\frac{\cos\Theta-x}{\sqrt{(\cos\Theta-x)^2+\sin^2\Theta}}+1\right]\right\}^{-1}, \tag{32}$$

which was found to agree with the results in [1, 7] (in [7] a search for an exact solution only at an angular point was examined in a somewhat different way, and the agreement was natural since the singularity of the initial integral equation as $x \to 0$ was by-passed). The effective degree of blackness of such a cavity is:

$$\varepsilon_{\text{eff}}=\varepsilon\left\{1+\frac{(1-\varepsilon)\cos\frac{\Theta}{2}}{\sqrt{\varepsilon}}\left[\operatorname{arctg}\left[\frac{(1+\varepsilon\operatorname{tg}\left(\frac{\pi}{4}-\frac{\Theta}{2}\right)+\varepsilon-1}{2\sqrt{\varepsilon}}\right]-\operatorname{arctg}\left[-\frac{(1+\varepsilon)\operatorname{tg}\frac{\Theta}{4}+\varepsilon-1}{2\sqrt{\varepsilon}}\right]\right]\right\}. \tag{33}$$

Figure 4 presents the results of the calculations for ε_{eff}.

In conclusion, it should be noted that although the singularity of the integral equations we have pointed out is not strictly observed for all forms of bodies, the physical basis of our approach is on firmer grounds, for bodies of finite dimensions, than the "balance" method used in [5]. In contrast to the latter, our approach has the possibility of directly obtaining analytical expressions for the distributions of thermal flows over the surface of bodies as a function of dimensionless variables without having to solve algebraic systems of high order. Also, taking into account the fact that the physical formulation itself is, generally speaking, approximate, one can expect that our approach will find application in engineering practice.

NOTATION

ε, emissivity; σ, Stefan–Boltzmann constant; T, temperature; ξ, η, strip coordinates; L, strip width; d, distance between strips; x, y, γ, dimensionless coordinates and distance; E_{eff}, effective radiant heat flux; β, dimensionless effective radiant heat flux; K, kernel of the integral equation; q, heat flux carried away; ρ, reflection coefficient; δ, delta-function; Q, total heat flux; Θ, opening angle of wedge-shaped cavity; i, number of body; E_{in}, incident radiant heat flux; T_p, equilibrium temperature; ε_{eff}, effective emissivity.

LITERATURE CITED

1. Sparrow, Gregg, Shell, and Manus, Teploperedacha, 83, No. 2, 136-146 (1961).
2. Sparrow and Grett, Teploperedacha, 83, No. 4, 130-140 (1961).
3. M. I. Otsisik, Compound Heat Exchange [in Russian], Moscow (1976).
4. B. P. Demidovich, I. A. Maron, and É. Z. Shuvalova, Numerical Methods of Analysis [in Russian], Moscow (1963).
5. G. L. Polyak, Izv. Energ. Inst. im. Krizhizhanovskogo, 3, 53-75 (1935).
6. D. Ivanenko and A. Sokolov, Classical Field Theory [in Russian], Moscow (1951).
7. Rasmussen and Jischke, AIAA J., 8, No. 7, 1360-1361 (1970).
8. Spacecraft Thermal Control Design Data, National Technical Information Service, 1, No. 76-20197, 372 (1975).